Magnetic Moments of Δ Baryons in Light Cone QCD Sum Rules

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Abstract

We calculate the magnetic moments of Δ baryons within the framework of QCD sum rules. A comparison of our results on the magnetic moments of the Δ baryons with the predictions of different approaches is presented.

1 Introduction

The extraction of the fundamental parameters of hadrons from experimental data requires some information about physics at large distances and they can not be calculated directly from fundamental QCD Lagrangian because at large distance strong coupling constant, α_s, becomes large and perturbation theory is invalid. For this reason for determination of hadron parameters, a reliable non-perturbative approach is needed. Among other non-perturbative approaches, QCD sum rules [1] is an especially powerful method in studying the properties of low-lying hadrons. In this method, deep connection between the hadron parameters and the QCD vacuum structure is established via a few condensate parameters. This method is adopted and extended in many works (see for example Refs. [2, 3, 4] and references therein). One of characteristic parameters of the hadrons is their magnetic moments. Calculation of the nucleon magnetic moments in the framework of QCD sum rules method using external fields technique, first suggested in [5], was carried out in [6, 7]. They were later refined and extended to the entire baryon octet in [8, 9].

Magnetic moments of the decuplet baryons are calculated in [10, 11] within the framework of QCD sum rules using external field. Note that in [10], from the decuplet baryons, only the magnetic moments of Δ^{++} and Ω^- were calculated. At present, the magnetic moments of Δ^{++} [12], Δ^0 [13] and Ω^- [14] are known from experiments. The experimental information provides new incentives for theoretical scrutiny of these physical quantities.

In this letter, we present an independent calculation of the magnetic moments of Δ^{++}, Δ^+, Δ^0, and Δ^- within the framework of an alternative approach to the traditional sum rules, i.e. the light cone QCD sum rules (LCQSR). Comparison of the predictions of this approach on magnetic moments with the results of other methods existing in the literature, and the experimental results is also presented.

The LCQSR is based on the operator product expansion on the light cone, which is an expansion over the twists of the operators rather than dimensions as in the traditional QCD sum rules. The main contribution comes from the lower twist operator. The matrix elements of the nonlocal operators between the vacuum and hadronic state defines the hadronic wave functions. (More about this method and its applications can be found in [15, 16] and references therein). Note that magnetic moments of the nucleon using LCQSR approach was studied in [17].

The paper is organized as follows. In Sect. II, the light cone QCD sum

rules for the magnetic moments of Δ^{++}, Δ^{+}, Δ^{0}, and Δ^{-} are derived. In Sect. III, we present our numerical analysis and conclusion.

2 Sum Rules for the Magnetic Moments of Δ baryons

According to the QCD sum rules philosophy, a quantitative estimate for the Δ magnetic moment can be obtained by equating two different representations of the corresponding correlator, written in terms of hadrons and quark-gluons. For this aim, we consider the following correlation function

$$\Pi_{\mu\nu} = i \int dx e^{ipx} \langle 0|\mathcal{T}\eta_\mu^B(x)\bar{\eta}_\nu^B(0)|0\rangle_\gamma \,, \tag{1}$$

where $\mathcal{T}$ is the time ordering operator, γ means external electromagnetic field. In this expression the η_μ^B's are the interpolating currents for the baryon B. This correlation function can be calculated on one side phenomenologically, in terms of the hadron properties and on the other side by the operator product expansion (OPE) in the deep Eucledian region of the correlator momentum $p^2 \to -\infty$ using QCD degrees of freedom. By equating both expressions, we construct the corresponding sum rules.

On the phenomenological side, by inserting a complete set of one hadron states into the correlation function, Eq. (1), one obtains:

$$\Pi_{\mu\nu}(p_1^2,p_2^2) = \sum_{B_1,\,B_2} \frac{\langle 0|\eta_\mu^B|B_1(p_1)\rangle}{p_1^2 - M_1^2} \langle B_1(p_1)|B_2(p_2)\rangle_\gamma \frac{\langle B_2(p_2)|\eta_\nu^B|0\rangle}{p_2^2 - M_2^2}, \tag{2}$$

where $p_2 = p_1 + q$, q is the photon momentum, B_i form a complete set of baryons having the same quantum numbers as B, with masses M_i.

The matrix elements of the interpolating currents between the ground state and the state containing a single baryon, B, with momentum p and having spin s is defined as:

$$\langle 0|\eta_\mu|B(p,s)\rangle = \lambda_B u_\mu(p,s)\,, \tag{3}$$

where λ_B is a phenomenological constant parametrizing the coupling strength of the baryon to the current, and u_μ is the Rarita-Schwinger spin-vector satisfying $(\not{p} - M_B)u_\mu = 0$, $\gamma_\mu u_\mu = p_\mu u_\mu = 0$. (For a discussion of the

properties of the Rarita-Schwinger spin-vector see e.g. [18]). In order to write down the phenomenological part of the sum rules, one also needs an expression for the matrix element $\langle B(p_1)|B(p_2)\rangle_\gamma$. In the general case, the electromagnetic vertex of spin 3/2 baryons can be written as

$$\langle B(p_1)|B(p_2)\rangle_\gamma = \epsilon_\rho \bar{u}_\mu(p_1)\mathcal{O}^{\mu\rho\nu}(p_1,p_2)u_\nu(p_2)\,, \tag{4}$$

where ϵ_ρ is the polarization vector of the photon and the Lorentz tensor $\mathcal{O}^{\mu\rho\nu}$ is given by:

$$\begin{aligned}\mathcal{O}^{\mu\rho\nu}(p_1,p_2) &= -g^{\mu\nu}\left[\gamma_\rho(f_1+f_2)+\frac{(p_1+p_2)_\rho}{2M_B}f_2+q_\rho f_3\right]- \\ &- \frac{q_\mu q_\nu}{(2M_B)^2}\left[\gamma_\rho(g_1+g_2)+\frac{(p_1+p_2)_\rho}{2M_B}g_2+q_\rho g_3\right]\end{aligned} \tag{5}$$

where the form factors f_i and g_i are (in the general case) functions of $q^2 = (p_1-p_2)^2$. In our problem, the values of the formfactors only at one point, $q^2=0$, are needed.

In our calculation, we also performed summation over spins of the Rarita-Schwinger spin vector,

$$\sum_s u_\sigma(p,s)\bar{u}_\tau(p,s) = -\frac{(\not{p}+M_B)}{2M_B}\left\{g_{\sigma\tau}-\frac{1}{3}\gamma_\sigma\gamma_\tau-\frac{2p_\sigma p_\tau}{3M_B^2}+\frac{p_\sigma\gamma_\tau-p_\tau\gamma_\sigma}{3M_B}\right\} \tag{6}$$

Using Eqs. (3-6), the correlation function can be expressed as the sum of various structures, not all of them independent. To remove the dependencies, an ordering of the gamma matrices should be chosen. For this purpose the structure $\gamma_\mu \not{p}_1 \not{\epsilon} \not{p}_2 \gamma_\nu$ is chosen. With this ordering, the correlation function becomes:

$$\begin{aligned}\Pi_{\mu\nu} &= \lambda_B^2\frac{1}{(p_1^2-M_B^2)(p_2^2-M_B^2)}\left[g_{\mu\nu}\ \not{p}_1 \not{\epsilon} \not{p}_2 \frac{g_M}{3}+\right. \\ &+ \left.\textit{other structures with } \gamma_\mu \textit{ at the beginning and } \gamma_\nu \textit{ at the end}\right]\end{aligned} \tag{7}$$

where g_M is the magnetic form factor, $g_M/3 = f_1+f_2$. g_M evaluated at $q^2=0$ gives the magnetic moment of the baryon in units of its natural magneton, $e\hbar/2m_Bc$. The appearance of the factor 3 can be understood from the fact that in the nonrelativistic limit, the maximum energy of the baryon in the

presence of a uniform magnetic field with magnitude H is $3(f_1+f_2)H \equiv g_M\, H$ [19].

The reason for choosing this structure can be explained as follows. In general the interpolation current might also have a non-zero overlap with spin $\frac{1}{2}$ baryons, but spin $\frac{1}{2}$ baryons do not contribute to the structure $g_{\mu\nu}\, \not{p}_1 \not{\varepsilon}\, \not{p}_2$ since their overlap is given by:

$$\langle 0|\eta_\mu|J=1/2\rangle = (Ap_\mu + B\gamma_\mu)u(p) \tag{8}$$

where $(\not{p}-m)u(p)=0$ and $(Am+4B)=0$ [19, 20].

In order to calculate the correlator (1) from the QCD side, first, appropriate currents should be chosen. For the case of the Δ baryons, they can be chosen as (see for example [11]):

$$\begin{aligned}
\eta_\mu^{\Delta^{++}} &= \epsilon^{abc}(u^{aT}C\gamma_\alpha u^b)u^c\,,\\
\eta_\mu^{\Delta^{+}} &= \frac{1}{\sqrt{3}}\epsilon^{abc}[2(u^{aT}C\gamma_\alpha d^b)u^c + (u^{aT}C\gamma_\alpha u^b)d^c]\,,\\
\eta_\mu^{\Delta^{0}} &= \frac{1}{\sqrt{3}}\epsilon^{abc}[2(d^{aT}C\gamma_\alpha u^b)d^c + (d^{aT}C\gamma_\alpha d^b)u^c]\,,\\
\eta_\mu^{\Delta^{-}} &= \epsilon^{abc}(u^{aT}C\gamma_\alpha u^b)u^c\,,
\end{aligned} \tag{9}$$

where C is the charge conjugation operator, a, b, c are color indices. It should be noted that these baryon currents are not unique, one can choose an infinite number of currents with the Δ baryon quantum numbers [4, 21].

On the QCD side, for the same correlation functions we obtain:

$$\begin{aligned}
\Pi_{\mu\nu}^{\Delta^{++}} = \quad & \Pi_{\mu\nu}^{\prime\Delta^{++}} + \frac{1}{2}\epsilon^{abc}\epsilon^{def}\int d^4x e^{ipx}\langle\gamma(q)|\bar{u}^f A_i u^a\\
& \Big\{2S_u^{cd}\gamma_\nu S_u^{\prime be}\gamma_\mu A_i + 2S_u^{cd}\gamma_\nu A_i^\prime\gamma_\mu S_u^{be} +\\
& +2A_i\gamma_\nu S_u^{\prime cd}\gamma_\mu S_u^{be} + S_u^{cd}\mathrm{Tr}(\gamma_\nu S_u^{\prime\,be}\gamma_\mu A_i) +\\
& +S_u^{cd}\mathrm{Tr}(\gamma_\nu A_i^\prime\gamma_\mu S_u^{be}) + A_i\mathrm{Tr}(\gamma_\nu S_u^{\prime\,cd}\gamma_\mu S_u^{be})\Big\}\,|0\rangle \qquad (10)\\
\Pi_{\mu\nu}^{\Delta^{+}} = \quad & \Pi_{\mu\nu}^{\prime\Delta^{+}} - \frac{1}{6}\epsilon^{abc}\epsilon^{def}\int d^4x e^{ipx}\langle\gamma(q)|\bar{u}^d A_i u^a\\
& \Big\{2A_i\gamma_\nu S_d^{\prime\,be}\gamma_\mu S_u^{cf} + 2A_i\gamma_\nu S_u^{\prime\,cf}\gamma_\mu S_d^{be} +\\
& +2S_d^{be}\gamma_\nu A_i^\prime\gamma_\mu S_u^{cf} + 2A_i\mathrm{Tr}(\gamma_\nu S_u^{\prime\,cf}\gamma_\mu S_d^{be}) +\\
& +S_d^{be}\mathrm{Tr}(\gamma_\nu A_i^\prime\gamma_\mu S_u^{cf}) +
\end{aligned}$$

$$
\begin{aligned}
&+2S_u^{cf}\gamma_\nu S_d'^{\,be}\gamma_\mu A_i + 2S_u^{cf}\gamma_\nu A_i'\gamma_\mu S_d^{be}+\\
&+2S_d^{be}\gamma_\nu S_u'^{\,cf}\gamma_\mu A_i + 2S_u^{cf}\mathrm{Tr}(\gamma_\nu A_i'\gamma_\mu S_d^{be})+\\
&+S_d^{be}\mathrm{Tr}(\gamma_\nu S_u'^{\,cf}\gamma_\mu A_i)\Big\} + \bar{d}^e A_i d^b\\
&\Big\{2S_u^{ad}\gamma_\nu A_i'\gamma_\mu S_u^{cf} + 2S_u^{ad}\gamma_\nu S_u'^{\,cf}\gamma_\mu A_i+\\
&+2A_i\gamma_\nu S_u'^{\,ad}\gamma_\mu S_u^{cf} + 2S_u^{ad}\mathrm{Tr}(\gamma_\nu S_u'^{\,cf}\gamma_\mu A_i)+\\
&+A_i\mathrm{Tr}(\gamma_\nu S_u'^{\,ad}\gamma_\mu S_u^{ad})\Big\}\,|0\rangle \qquad (11)
\end{aligned}
$$

where $A_i = 1$, γ_α, $\sigma_{\alpha\beta}/\sqrt{2}$, $i\gamma_\alpha\gamma_5$, γ_5, a sum over A_i implied, $S' \equiv CS^TC$, $A_i' = CA_i^TC$, with T denoting the transpose of the matrix, and S_q is the full light quark propagator with both perturbative and non-perturbative contributions:

$$
\begin{aligned}
S_q &= \langle 0|\mathcal{T}\bar{q}(x)q(0)|0\rangle\\
&= \frac{i\not{x}}{2\pi^2x^4} - \frac{\langle\bar{q}q\rangle}{12} - \frac{x^2}{192}m_0^2\langle\bar{q}q\rangle -\\
&- ig_s\int_0^1 dv\left[\frac{\not{x}}{16\pi^2x^2}G_{\mu\nu}(vx)\sigma_{\mu\nu} - vx_\mu G_{\mu\nu}(vx)\gamma_\nu\frac{i}{4\pi^2x^2}\right] \qquad (12)
\end{aligned}
$$

The $\Pi'^{\Delta}_{\mu\nu}$s in Eqs. (10) and (11) describe diagrams in which the photon interact with the quark lines perturbatively. Their explicit expressions can be obtained from the remaining terms by substituting all occurances of

$$
\bar{q}^a(x)A_iq^bA_{i\alpha\beta} \to 2\left(\int d^4yF_{\mu\nu}y_\nu S_q^{pert}(x-y)\gamma_\mu S_q^{pert}(y)\right)^{ba}_{\alpha\beta} \qquad (13)
$$

where the Fock-Schwinger gauge is used, and S_q^{pert} is the perturbative quark propogator, i.e. the first term in Eq. (12).

The corresponding expressions for the correlation functions for the Δ^- and Δ^0 baryons can be obtained from Eqs. (10) and (11), if one exchanges u-quarks by d-quarks and vice versa, respectively.

In order to be able to calculate the QCD part of the sum rules, one needs to know the matrix elements $\langle\gamma(q)|\bar{q}A_iq|0\rangle$. Upto twist-4, the non-zero matrix elements given in terms of the photon wave functions are [22-24]:

$$
\begin{aligned}
\langle\gamma(q)|\bar{q}\gamma_\alpha\gamma_5q|0\rangle &= \frac{f}{4}e_q\epsilon_{\alpha\beta\rho\sigma}\epsilon^\beta q^\rho x^\sigma\int_0^1 due^{iuqx}\psi(u)\\
\langle\gamma(q)|\bar{q}\sigma_{\alpha\beta}q|0\rangle &= ie_q\langle\bar{q}q\rangle\int_0^1 due^{iuqx}
\end{aligned}
$$

$$
\begin{aligned}
\times & \quad \{(\epsilon_\alpha q_\beta - \epsilon_\beta q_\alpha)[\chi\phi(u) + x^2[g_1(u) - g_2(u)]] \\
+ & \quad [qx(\epsilon_\alpha x_\beta - \epsilon_\beta x_\alpha) + \epsilon x(x_\alpha q_\beta - x_\beta q_\alpha)]\, g_2(u)\} \quad (14)
\end{aligned}
$$

where χ is the magnetic susceptibility of the quark condensate and e_q is the quark charge. The functions $\phi(u)$ and $\psi(u)$ are the leading twist-2 photon wave functions, while $g_1(u)$ and $g_2(u)$ are the twist-4 functions.

Note that, since we have assumed massless quarks, $m_u = m_d = 0$, and exact SU(2) flavor symmetry, which implies $\langle \bar{u}u\rangle = \langle \bar{d}d\rangle$, the u and d quark propagators are identical, $S_u = S_d$, whereas for the wave functions, the only difference is due to the different charges of the two quarks. The general expressions given by Eqs. (10) and (11), under exact $SU(2)$ flavor symmetry lead to the following results:

$$
\begin{aligned}
\Pi_{\mu\nu}^{\Delta^{++}} &= -\frac{1}{2} e_u \mathcal{C} \\
\Pi_{\mu\nu}^{\Delta^{+}} &= -\frac{1}{6}(2e_u + e_d)\mathcal{C} \\
\Pi_{\mu\nu}^{\Delta^{0}} &= -\frac{1}{6}(2e_d + e_u)\mathcal{C} \\
\Pi_{\mu\nu}^{\Delta^{-}} &= -\frac{1}{2} e_d \mathcal{C} \qquad (15)
\end{aligned}
$$

where $\mathcal{C}$ is a factor independent of the quark charges. From Eq. (15), the following exact relations between theoretical parts of the correlator functions immediately follows:

$$
\begin{aligned}
\Pi_{\mu\nu}^{\Delta^{+}} &= -\Pi_{\mu\nu}^{\Delta^{-}} = \frac{1}{2}\Pi_{\mu\nu}^{\Delta^{++}} \\
\Pi_{\mu\nu}^{\Delta^{0}} &= 0 \qquad (16)
\end{aligned}
$$

Hence, from now on, only the results for Δ^{++} will be reported and for the other Δ's, the corresponding results can be obtained from the Eqs. (16). Using Eqs. (12) and (14), from Eq. (10) and after some algebra and after performing Fourier transformation, for the coefficient of the structure $g_{\mu\nu}\ \not{p}_1\ \not{\epsilon}\ \not{p}_2$, we get:

$$
\begin{aligned}
\Pi &= e_u \int_0^1 du \left\{ \frac{f\psi(u)}{48\pi^2}\left[4\ln(-P^2) + \frac{\langle g^2G^2\rangle}{12P^4}\right] + \right. \\
& + \frac{8}{3P^4}\langle \bar{u}u\rangle^2[g_1(u) - g_2(u)] + \frac{\chi\phi(u)\langle \bar{u}u\rangle^2}{6P^2}\left(\frac{m_0^2}{P^2} + 4\right) +
\end{aligned}
$$

$$+ \frac{2\langle\bar{u}u\rangle^2}{3P^4} - \frac{\langle g^2G^2\rangle}{768\pi^4 P^2} - \frac{3P^2\ln(-P^2)}{64\pi^4}\Bigg\} \tag{17}$$

where $P = p + qu$. In Eq. (17), polynomials in P^2 are omitted since they do not contribute after the Borel transformation.

As stated earlier, in order to obtain the sum rules, one equates the phenomenological and theoretical expressions obtained within QCD. After performing the Borel transformation on the variables p^2 and $(p+q)^2$ in order to suppress the contributions of the higher resonances and the continuum, the following sum rules is obtained for the magnetic moment of Δ^{++}:

$$\begin{aligned} g_M &= \frac{3e_u}{\lambda_\Delta^2} e^{\frac{M_\Delta^2}{M^2}} \Bigg\{ \frac{f\psi(u_0)}{12\pi^2}\left[\frac{\langle g^2G^2\rangle}{48} - M^4 f_1(\frac{s_0}{M^2})\right] + \\ &+ \frac{8}{3}\langle\bar{u}u\rangle^2[g_1(u_0) - g_2(u_0)] + \\ &+ \frac{\chi\phi(u_0)\langle\bar{u}u\rangle^2}{6}\left[m_0^2 - 4M^2 f_0(\frac{s_0}{M^2})\right] \\ &\quad + \frac{2\langle\bar{u}u\rangle^2}{3} + \frac{\langle g^2G^2\rangle M^2}{768\pi^4} f_0(\frac{s_0}{M^2}) + \frac{3M^6}{64\pi^4} f_2(\frac{s_0}{M^2})\Bigg\} \end{aligned} \tag{18}$$

where the functions

$$f_n(x) = 1 - e^{-x}\sum_{k=0}^{n}\frac{x^k}{k!} \tag{19}$$

are used to subtract the contributions of the continuum. In Eq. (18), s_0 is the continuum threshold,

$$\begin{aligned} u_0 &= \frac{M_1^2}{M_1^2 + M_2^2} \\ \frac{1}{M^2} &= \frac{1}{M_1^2} + \frac{1}{M_2^2} \end{aligned}$$

As we are dealing with just a single baryon, the Borel parameters M_1^2 and M_2^2 can be taken to be equal, i.e. $M_1^2 = M_2^2$, from which it follows that $u_0 = 1/2$.

3 Numerical Analysis

From Eq. (18), one sees that, in order to calculate the numerical value of the magnetic moment of the Δ^{++}, besides several numerical constants, one

requires expressions for the photon wave functions. It was shown in [22, 23] that the leading photon wave functions receive only small corrections from the higher conformal spin, so they do not deviate much from the asymptotic form. Following [23, 24], we shall use the following photon wave functions:

$$
\begin{aligned}
\phi(u) &= 6u\bar{u} \\
\psi(u) &= 1 \\
g_1(u) &= -\frac{1}{8}\bar{u}(3-u) \\
g_2(u) &= -\frac{1}{4}\bar{u}^2
\end{aligned}
$$

where $\bar{u} = 1 - u$. The values of the other constants that are used in the calculation are: $f = 0.028\,GeV^2$, $\chi = -4.4\,GeV^{-2}$ [25] (in [26], χ is estimated to be $\chi = -3.3\,GeV^{-2}$), $\langle g^2G^2\rangle = 0.474\,GeV^4$, $\langle\bar{u}u\rangle = -(0.243)^3\,GeV^3$, $m_0^2 = (0.8 \pm 0.2)\,GeV^2$ [27], $\lambda_\Delta = 0.038$ [28].

Having fixed the input parameters, our next task is to find a region of Borel parameter, M^2, where dependence of the magnetic moments on M^2 and the continuum threshold s_0 is rather weak and at the same time the higher dimension operators, higher states and continuum contributions remain under control. We demand that these contributions are less then 35%. Under this requirement, the working region for the Borel parameter, M^2, is found to be $1.1\,GeV^2 \leq M^2 \leq 1.4\,GeV^2$. Finally, in this range of the Borel parameter, the magnetic moment of Δ^{++} is found to be $(4.55 \pm 0.03)\,\mu_N$. This prediction on the magnetic moment is obtained at $s_0 = 4.4\,GeV^2$. Choosing $s_0 = 3.8\,GeV^2$ or $s_0 = 4.2\,GeV^2$ changes the result at most by 6%(see Fig. 1). The calculated magnetic moment is in good agreement with the experimental result $(4.52 \pm 0.50 \pm 0.45)\,\mu_N$ [14]. Our results on the magnetic moments for Δ^+, Δ^0 and Δ^- are presented in Table 1. For completeness, in this table, we have also presented the predictions of other approaches. Comparing the values presented in Table 1, it is seen that our predictions on magnetic moments is larger than the QCDSR predictions.

Finally, for the calculation of the magnetic moments of other members of the decuplet (which contain at least one s-quark), the correction due to the strange quark mass should be taken into account. Their calculations would be presented in a future work.

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Table 1: Comparisons of Δ baryon magnetic moments from various calculations: this work (LCQSR),QCDSR [11] lattice QCD (Latt) [29], chiral perturbation theory (χPT) [30], light-cone relativistic quark model (RQM) [31], non-relativistic quark model (NQM) [32], chiral quark-soliton model (χQSM) [33], chiral bag-model (χB) [34]. All results are in units of nuclear magnetons.

Baryon	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}
Exp.	4.5±1.0		~ 0	
LCQCD	4.4±0.8	2.2±0.4	0.00	-2.2±0.4
QCDSR	4.13±1.30	2.07±0.65	0.00	-2.07±0.65
Latt.	4.91±0.61	2.46±0.31	0.00	-2.46±0.31
χPT	4.0±0.4	2.1±0.2	-0.17±0.04	-2.25±0.25
RQM	4.76	2.38	0.00	-2.38
NQM	5.56	2.73	-0.09	-2.92
χQSM	4.73	2.19	-0.35	-2.90
χB	3.59	0.75	-2.09	-1.93

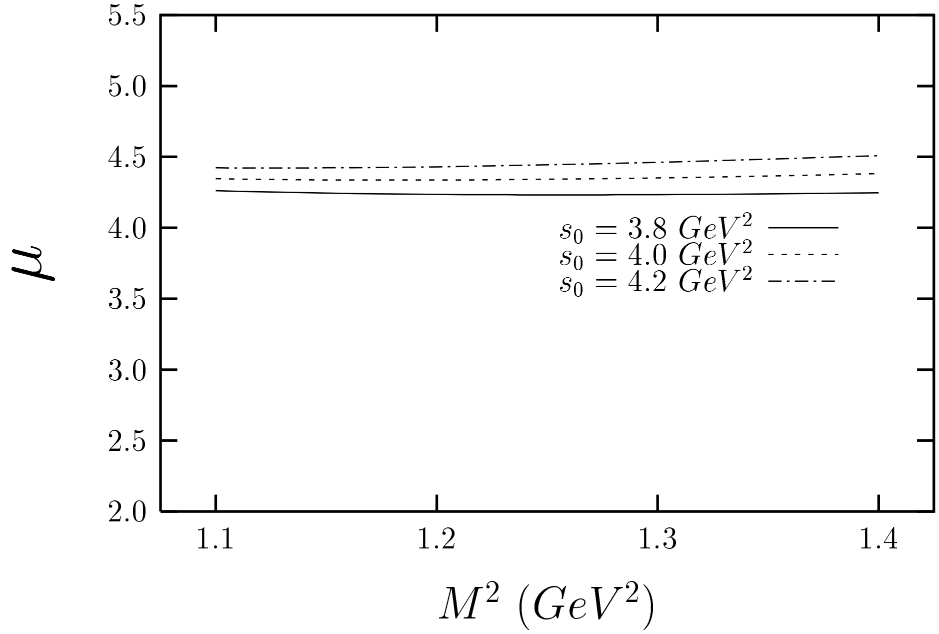

Figure 1: The dependence of the magnetic moment of Δ^{++} on the borel parameter M^2 (in nuclear magneton units) for three different values of the continuum threshold, s_0.